

Assignment III: MTH 213, Fall 2017

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QUESTION 1. Let X be number of defective computers. Given : a) X is an divisible by 6, b) $X \equiv 9 \pmod{15}$, and $X \equiv 7 \pmod{11}$. Find X if $330 \leq X \leq 660$ [Note that X is divisible by 3 means $X \equiv 0 \pmod{3}$]

We have $X \equiv 0 \pmod{6}$, $X \equiv 9 \pmod{15}$, $X \equiv 7 \pmod{11}$. Since $\gcd(6, 15) = 3$. We need to get rid of the factor 3 from 15 or 6. Note that $X \equiv 0 \pmod{6}$ implies $X \equiv 0 \pmod{3}$. Also $X \equiv 9 \pmod{15}$ implies $X \equiv 9 \pmod{3}$ and hence implies $X \equiv 0 \pmod{3}$. So you may remove the factor 3 from 6 or from 15.

New system: SOLVE $X \equiv 0 \pmod{6}$, $X \equiv 4 \pmod{5}$, $X \equiv 6 \pmod{11}$ [Here we removed 3 from 15, Since $9 \pmod{5} = 4$, $X \equiv 9 \pmod{5}$ is the same as $X \equiv 4 \pmod{5}$].

OR Solve $X \equiv 0 \pmod{2}$, $X \equiv 9 \pmod{15}$, $X \equiv 6 \pmod{11}$ [Here we removed 3 from 6]
Either one should give you the same solution : $204 + 330 = 534$.

QUESTION 2. (i) Add $(7AC43)_{16} + (29B)_{16}$

(ii) Subtract $(7854)_9 - (1428)_9$

(iii) multiply $(234)_5 \cdot (42)_5$

(iv) multiply $(A6B)_{16} \cdot (9A)_{16}$

QUESTION 3. Solve over Z : $x \equiv 7 \pmod{8}$, $x \equiv 1 \pmod{6}$, and $x \equiv 4 \pmod{5}$ the number here is 5

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See comments on Question one: $\gcd(8, 6) = 2$. So we need to get rid of the factor 2 from 6 or 8. Note $x \equiv 7 \pmod{8}$ implies $x \equiv 7 \pmod{2}$ implies $x \equiv 1 \pmod{2}$ [Since $7 \pmod{2}$ is 1]. Now $x \equiv 1 \pmod{6}$ implies $x \equiv 1 \pmod{2}$.

So SOLVE $x \equiv 7 \pmod{8}$, $x \equiv 1 \pmod{3}$, $x \equiv 4 \pmod{5}$ [Here we removed the factor 2 from 6]

NOW if we remove the factor 2 from 8, we have $x \equiv 3 \pmod{4}$, $x \equiv 1 \pmod{6}$, $x \equiv 4 \pmod{5}$ and we cannot use the CRT. So we must stick with the first option. Solution $79 + 120n$, where n any integer.